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Solution of the gauge identities in the axial gauge†

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Abstract. Starting from the spectral representation of the two-point functions in the axial gauge, we 'solve' the gauge identities so as to express the higher-point Green functions *linearly* in terms of the two-point spectral function. The four-point functions are an important input for investigations of scalar electrodynamics and vector chromodynamics based on the gauge technique.

1. Introduction

There has been considerable interest in application of the gauge technique (Salam 1963, Delbourgo 1979a) over the last year or so, with the purpose of probing the ultraviolet and infrared structure of chromodynamics (Ball and Zachariasen 1978, Delbourgo 1979b, Bargadda 1979, Anishetty *et al* 1979). Because the technique is founded on a determination of the longitudinal charged line amplitudes in terms of the charged propagator, the use of the axial gauge is strongly indicated in such theories—other gauge choices lead to difficult identities involving fictitious particle amplitudes. Therefore, in chromodynamics, a significant part of the input is the solution of the gauge identities in the axial gauge (up to the usual unknown transverse pieces). In this paper we want to describe the general solution of this problem without resorting to any approximation. We shall determine the fully connected Green functions in terms of the spectral function of the charged line propagator in a form which is ready-made for insertion into the Dyson–Schwinger equation. Since the relation we obtain is linear, as is the equation for the Green function, the gauge technique is guaranteed to provide a linear equation for the spectral function even with the inclusion of intermediate three-gluon intermediate states. This approach differs markedly from that of Anishetty *et al* (1979) who solve the identities for one-particle irreducible amplitudes, do not make use of spectral representation and obtain complicated nonlinear equations for their propagator.

In § 2 we look at the simpler problem of scalar electrodynamics in the axial gauge; this bears some similarity to chromodynamics, in that the two-photon two-meson amplitude enters into the coupled field equations in a fundamental way. The full solution of gauge identities is spelled out in detail. We then repeat the exercise for spinor electrodynamics in § 3, although the four-point amplitude is not so basically important. With the experience gained from electrodynamics, we are able to tackle

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chromodynamics in § 4 and obtain the complete solution for the four-gluon Green function. The influence of this amplitude on the gauge technique investigation of the gluon propagator will be described in a separate publication.

2. Scalar electrodynamics

We begin with the spectral representation (Johnson 1960) of the charged meson propagator†

$$\Delta(p)=\int \mathrm{d}W^2\frac{\rho(W^2,p\,.\,n)}{p^2-W^2+\mathrm{i}0} \tag{1}$$

for the axial gauge fixed through $n\,.\,A=0$. It is convenient to take n as a unit vector along the temporal direction to simplify some of the writing below, although the final results do not, of course, depend on this choice—it is anyhow rather trivial to reinstate the n-dependence at the end. We have already described the solution of the identity

$$(p'-p)^{\mu}\Delta(p')\Gamma_{\mu}(p',p)\Delta(p)=\Delta(p)-\Delta(p') \tag{2}$$

in an earlier paper (Delbourgo 1978). It is simply

$$\begin{aligned}(\Delta\Gamma_\mu\Delta)(p',p)=&\tfrac{1}{2}\int \mathrm{d}W^2[\rho(W^2,p_0')+\rho(W^2,p_0)]\frac{1}{p'^2-W^2}(p+p')_\mu\frac{1}{p^2-W^2}\\ &+\tfrac{1}{2}\int \mathrm{d}W^2\frac{n_\mu}{(p'-p)_0}[\rho(W^2,p_0')-\rho(W^2,p_0)]\Big(\frac{1}{p'^2-W^2}+\frac{1}{p^2-W^2}\Big).\end{aligned} \tag{3}$$

Observe that the n_μ component here is irrelevant for the physics, since it vanishes upon contraction with an external polarisation vector or with the photon propagator $D^{\mu\nu}(p'-p)$. Note also the averaging over incoming and outgoing spectral functions in (3).

Our aim now is to solve the higher-point identities‡ (remember $p'+k'=p+k$)

$$k'^{\nu}(\Delta\Gamma_{\nu\mu}\Delta)(p',k';p,k)=(\Delta\Gamma_\mu\Delta)(p'+k',p)-(\Delta\Gamma_\mu\Delta)(p',p-k') \tag{4a}$$

$$k^{\mu}(\Delta\Gamma_{\nu\mu}\Delta)(p',k';p,k)=(\Delta\Gamma_\nu\Delta)(p',p+k)-(\Delta\Gamma_\nu\Delta)(p'-k,p) \tag{4b}$$

etc, using the form (3) so as to find the connected (but not one-particle irreducible) amplitudes $\Delta\Gamma\Delta$ in terms of appropriate integrals over ρ. (Figure 1 gives a diagrammatic representation of the identities (4).) To understand the steps, it is very helpful to be conversant with the kinematical solution provided by lowest-order perturbation theory.

† The bare propagator is retrieved by setting $\rho(W^2,p\,.\,n)=\delta(W^2-m^2)$. Because of charge conjugation symmetry $\Delta(p)=\Delta(-p)$, so ρ is even in $p\,.\,n$. First-order perturbation theory expressions for ρ can be found in Delbourgo and Phocas-Cosmetatos (1979), verifying the evenness.

‡ In (4), as in (2), there remain unknown transverse parts orthogonal to the momenta which are not determined by the gauge identities but only by the dynamics. For instance, in (2) the transverse vertex

$$\Delta(p')\Gamma^{\mathrm{T}}_{\mu}(p',p)\Delta(p)=[(p'-p)^2\eta_{\mu\nu}-(p'-p)_\mu(p'-p)_\nu](p'+p)^\nu T[p^2,p'^2,(p-p')^2]$$

makes no contribution, and can only be evaluated by going to second gauge approximation in (4).

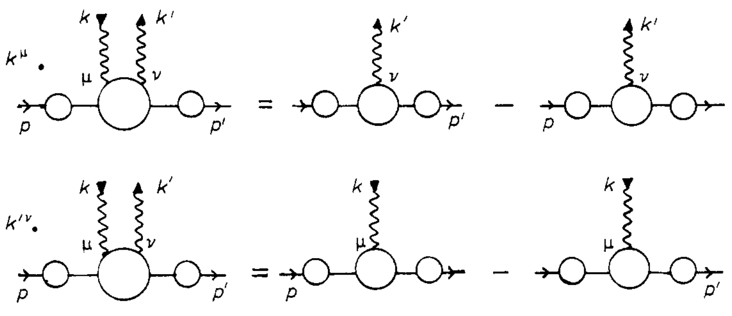

Figure 1. Gauge identities in electrodynamics. The solid line stands for the charged particle, meson or fermion.

For example, the kinematic identity

$$\frac{1}{(p'+k')^2-W^2}(p'+p+k')_\mu \frac{1}{p^2-W^2} - \frac{1}{p'^2-W^2}(p+p'-k')_\mu \frac{1}{(p-k')^2-W^2}$$
$$= \frac{1}{p'^2-W^2}\left(2k'_\mu - \frac{k'\,.\,(2p+k')}{(p'+k')^2-W^2}(2p+k)_\mu - (2p'-k)_\mu \frac{k'\,.\,(2p-k')}{(p-k')^2-W^2}\right)$$
$$\times \frac{1}{p^2-W^2} \tag{5}$$

represents the satisfaction of (4a) to order e^2. Upon substitution of (3), the right-hand side of (4a) is basically of the form (5) (a linear superposition thereof), except for the complication of the energy dependence within the spectral function ρ. The latter difficulty can be overcome by taking sums and differences over ρ, and making factorisations such as

$$\rho(W^2, p'_0+k'_0) - \rho(W^2, p'_0) = k'^\nu \frac{n_\nu}{k'\,.\,n}[\rho(W^2, p'_0+k'_0) - \rho(W^2, p'_0)].$$

The justification for this choice is that after factorisation there remains no singularity in the Green function $\Delta\Gamma\Delta$ as $k'\,.\,n \to 0$.

Having outlined the essential steps of the decomposition, let us quote the entire solution of (4) obtained by these manipulations:

$$\Delta(p')\Gamma_{\nu\mu}(p'k'; p, k)\Delta(p)$$
$$= \tfrac{1}{4}\int \mathrm{d}W^2[\rho(W^2, p'_0) + \rho(W^2, p'_0+k'_0) + \rho(W^2, p_0-k'_0) + \rho(W^2, p_0)]$$
$$\times \frac{1}{p'^2-W^2}\left(2\eta_{\nu\mu} - (2p'+k')_\nu \frac{1}{(p'+k')^2-W^2}(2p+k)_\mu\right.$$
$$\left. - (2p'-k)_\mu \frac{1}{(p'-k)^2-W^2}(2p-k')_\nu\right)\frac{1}{p^2-W^2}$$
$$+ \tfrac{1}{4}\int \mathrm{d}W^2 \frac{n_\nu}{k'_0}[\rho(W^2, p'_0+k'_0) - \rho(W^2, p'_0) + \rho(W^2, p_0) - \rho(W^2, p_0-k'_0)]$$

$$\times\left(\frac{1}{(p+k)^2-W^2}(2p+k)_\mu\frac{1}{p^2-W^2}+\frac{1}{p'^2-W^2}(2p'-k)_\mu\frac{1}{(p'-k)^2-W^2}\right)$$

$$+\tfrac{1}{4}\int \mathrm{d}W^2\frac{n_\mu}{k_0}[\rho(W^2,p_0+k_0)-\rho(W^2,p_0)+\rho(W^2,p_0')-\rho(W^2,p_0'-k_0)]$$

$$\times\left(\frac{1}{(p'+k')^2-W^2}(2p'+k')_\nu\frac{1}{p'^2-W^2}+\frac{1}{(p-k')^2-W^2}(2p-k')_\nu\frac{1}{p^2-W^2}\right)$$

$$+\tfrac{1}{4}\int \mathrm{d}W^2\frac{n_\mu n_\nu}{k_0k_0'}[\rho(W^2,p_0'+k_0')-\rho(W^2,p_0)-\rho(W^2,p_0')$$

$$+\rho(W^2,p_0-k_0')]$$

$$\times\left(\frac{1}{p'^2-W^2}+\frac{1}{(p+k)^2-W^2}+\frac{1}{(p-k')^2-W^2}+\frac{1}{p^2-W^2}\right). \tag{6}$$

The symmetry of the function under the interchange $p\leftrightarrow -p'$ and $k,\mu\leftrightarrow -k',\nu$, as required by Bose statistics, is easily verified.

It is worth reiterating that the full expression (6) is not needed in practice: the complete Green function, including photon propagators D, simplifies to

$$\Delta(p')D^{\nu\nu'}(k')\Gamma_{\nu'\mu'}(p',k';p,k)D^{\mu'\mu}(k)\Delta(p)$$

$$=\tfrac{1}{4}\int \mathrm{d}W^2[\rho(W^2,p'.n)+\rho(W^2,(p'+k').n)+\rho(W^2,(p-k').n)+\rho(W^2,p.n)]$$

$$\times\frac{D^{\nu\nu'}(k')}{p'^2-W^2}\left(2\eta_{\nu'\mu'}-(2p'+k')_{\nu'}\frac{1}{(p+k)^2-W^2}(2p+k)_{\mu'}\right.$$

$$\left.-(2p'-k)_{\mu'}\frac{1}{(p'-k)^2-W^2}(2p-k')_{\nu'}\right)\frac{D^{\mu'\mu}(k)}{p^2-W^2} \tag{7}$$

and this is simply a weighted integral over mass W of the classical amplitude for a charged scalar meson possessing that mass; the weighting includes an average over incoming, outgoing and intermediate charged line spectral functions. This structure is very important, and the results to be derived in spinor electrodynamics will turn out to be very similar.

Similarly, when amplitudes with additional photons are considered, the gauge identities will be satisfied by averaging the classical amplitudes over spectral functions associated with the external and all the internal (Born term) lines for every possible channel.

3. Spinor electrodynamics

We recall that in the axial gauge the spinor propagator has the general structure

$$S(p)=A(p^2,p.n)+\gamma.pB(p^2,p.n)+\gamma.n\,p.nC(p^2,p.n)$$

where A, B and C are scalar functions, even with respect to the argument $p.n$.

Correspondingly, we may write a Lehmann spectral form

$$S(p)=\int \frac{a(W^2, p\,.\,n)+\gamma\,.\,pb(W^2, p\,.\,n)+\gamma\,.\,n\,p\,.\,nc(W^2, p\,.\,n)}{p^2-W^2+\mathrm{i}0}\,\mathrm{d}W^2$$

or, better still,

$$S(p)=\left(\int_{-\infty}^{-m}+\int_{m}^{\infty}\right)\mathrm{d}W\frac{\rho(W, p\,.\,n)}{\gamma\,.\,p-W+\mathrm{i}0\epsilon(W)} \tag{8}$$

where

$$\rho(W, p\,.\,n)\equiv\epsilon(W)[W\rho_1(W^2, p\,.\,n)+\rho_2(W^2, p\,.\,n)+\gamma\,.\,n\,p\,.\,n\rho_3(W^2, p\,.\,n)]$$

is a matrix function of $\gamma\,.\,n$ and the ρ_i are even in $p\,.\,n$. In particular, because ρ_3 is a function of W^2 by C and P conservation, $\rho(W, p\,.\,n)$ effectively commutes with $\gamma\,.\,p$ in the denominator, so the order of numerator and denominator in (8), which might otherwise have concerned us, is irrelevant. The ρ_i have been calculated in first-order perturbation theory (Delbourgo and Phocas-Cosmetatos 1979).

The gauge identities are identical with (2) and (4) except, of course, that S replaces Δ. The analogue of (5) is

$$\begin{aligned}\frac{1}{\gamma\,.\,(p'+k')-W}\gamma_\mu\frac{1}{\gamma\,.\,p-W}&-\frac{1}{\gamma\,.\,p'-W}\gamma_\mu\frac{1}{\gamma\,.\,(p-k')-W}\\ &=-\frac{1}{\gamma\,.\,p'-W}\left(k'\,.\,\gamma\frac{1}{\gamma\,.\,(p'+k')-W}\gamma_\mu+\gamma_\mu\frac{1}{\gamma\,.\,(p-k')-W}k'\,.\,\gamma\right)\\ &\quad\times\frac{1}{\gamma\,.\,p-W}\end{aligned} \tag{9}$$

and we already possess (Delbourgo 1978) the solution of the vertex identity,

$$\begin{aligned}S(p')\Gamma_\mu(p', p)S(p)&\\ =\tfrac{1}{2}\int &\mathrm{d}W[\rho(W, p_0')+\rho(W, p_0)]\frac{1}{\gamma\,.\,p'-W}\gamma_\mu\frac{1}{\gamma\,.\,p-W}\\ +\tfrac{1}{2}\int &\mathrm{d}W\frac{n_\mu}{(p'-p)_0}[\rho(W, p_0)-\rho(W, p_0')]\left(\frac{1}{\gamma\,.\,p'-W}+\frac{1}{\gamma\,.\,p-W}\right).\end{aligned} \tag{10}$$

Therefore, after tracing out the parallel argument to the scalar case, one finishes up with the four-point answer,

$$\begin{aligned}S(p')\Gamma_{\nu\mu}(p', k'; p, k)S(p)&\\ =-\tfrac{1}{4}\int &\mathrm{d}W[\rho(W, p'\,.\,n)+\rho(W, (p'+k')\,.\,n)\\ &+\rho(W, (p-k')\,.\,n)+\rho(W, p\,.\,n)]\\ &\times\frac{1}{\gamma\,.\,p'-W}\left(\gamma_\nu\frac{1}{\gamma\,.\,(p+k)-W}\gamma_\mu+\gamma_\mu\frac{1}{\gamma\,.\,(p'-k)-W}\gamma_\nu\right)\frac{1}{\gamma\,.\,p-W}\end{aligned} \tag{11}$$

up to n_μ, n_ν terms which are very similar in form to (6) but disappear in all physical circumstances. Again one finds the classical Born graphs weighted by the spectral function averaged over all charged channels.

4. Chromodynamics

We are now fully prepared to attack the chromodynamic gauge identities. For the vector meson Green functions (figure 2 gives the pictorial description and notation) these read

$$p^\lambda \Delta^{\mu\mu'}(q)\Gamma^{abc}_{\lambda\mu'\nu'}(p,q,r)\Delta^{\nu'\nu}(r) = (\Delta^{\mu\nu}(r) - \Delta^{\mu\nu}(q))f^{abc} \tag{12}$$

$$\begin{aligned} &k^\kappa \Delta^{\lambda\lambda'}(p)\Delta^{\mu\mu'}(q)\Delta^{\nu\nu'}(r)\Gamma^{abcd}_{\kappa\lambda'\mu'\nu'}(k,p,q,r) \\ &\quad = f^{abe}\Delta^{\lambda\lambda'}(q+r)\Delta^{\mu\mu'}(q)\Delta^{\nu\nu'}(r)\Gamma^{ecd}_{\lambda'\mu'\nu'}(-q-r,q,r) \\ &\quad\quad + f^{ace}\Delta^{\mu\mu'}(r+p)\Delta^{\nu\nu'}(r)\Delta^{\lambda\lambda'}(p)\Gamma^{edb}_{\mu'\nu'\lambda'}(-r-p,r,p) \\ &\quad\quad + f^{ade}\Delta^{\nu\nu'}(p+q)\Delta^{\lambda\lambda'}(p)\Delta^{\mu\mu'}(q)\Gamma^{ebc}_{\nu'\lambda'\mu'}(-p-q,p,q) \end{aligned} \tag{13}$$

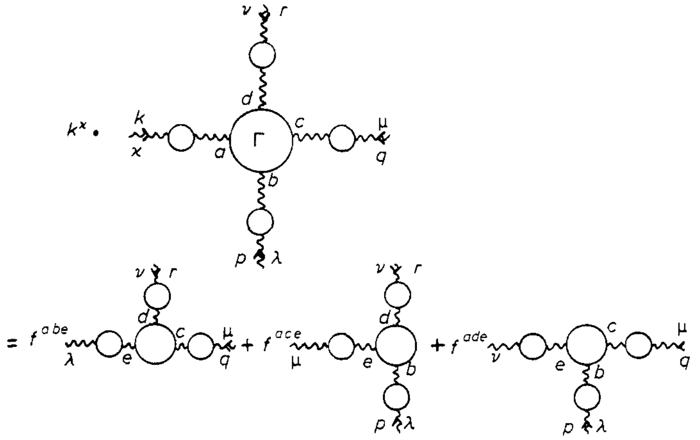

Figure 2. Gauge identities for connected Green functions in chromodynamics.

and are obviously satisfied in lowest-order perturbation theory when

$$\begin{aligned} \Gamma^{abc}_{\lambda\mu\nu}(p,q,r) &\to f^{abc}[(p-q)_\lambda\eta_{\mu\nu} + (q-r)_\mu\eta_{\nu\lambda} + (r-p)_\nu\eta_{\lambda\mu}] \\ &\equiv f^{abc}\Lambda(p,q,r)_{\lambda\mu\nu} \end{aligned} \tag{14}$$

and

$$\begin{aligned} \Gamma^{abcd}(k,p,q,r) \to\; & f^{abe}f^{cde}[(\eta_{\kappa\nu}\eta_{\lambda\mu} - \eta_{\kappa\mu}\eta_{\lambda\nu}) + \Lambda_{\kappa\lambda\rho}(k,p,\,)\Delta_0^{\rho\sigma}\Lambda_{\sigma\mu\nu}(\,,q,r)] \\ & + f^{ace}f^{dbe}[(\eta_{\kappa\lambda}\eta_{\mu\nu} - \eta_{\kappa\nu}\eta_{\mu\lambda}) + \Lambda_{\kappa\mu\rho}(k,q,\,)\Delta_0^{\rho\sigma}\Lambda_{\sigma\nu\lambda}(\,,r,p)] \\ & + f^{ade}f^{bce}[(\eta_{\kappa\mu}\eta_{\nu\lambda} - \eta_{\kappa\lambda}\eta_{\nu\mu}) + \Lambda_{\kappa\nu\rho}(k,r,\,)\Delta_0^{\rho\sigma}\Lambda_{\sigma\nu\lambda}(\,,p,q)] \end{aligned} \tag{15}$$

with

$$\Delta^0_{\mu\nu}(p) \equiv p^{-2}\left(-\eta_{\mu\nu} + \frac{p_\mu n_\nu + p_\nu n_\mu}{p\,.\,n} - \frac{p_\mu p_\nu n^2}{(p\,.\,n)^2}\right) \equiv \frac{\mathscr{A}_{\mu\nu}(p)}{p^2} \tag{16}$$

after factorising out a Kronecker delta in internal indices as in (12) and (13).

Although we are primarily interested in the connected Green functions, it is worth noting that the identities are very similar for the one-particle irreducible amplitudes, and involve the two-point function $\Delta^{-1\mu\nu}$, the three-point function $\Gamma_{\lambda\mu\nu}$, the four-point function $C_{\kappa\lambda\mu\nu}$, etc (see figure 3). Thus

$$p^{\lambda}\Gamma^{abc}_{\lambda MN}(p,q,r)=f^{abc}[\Delta^{-1}(q)-\Delta^{-1}(r)]_{MN} \tag{17}$$

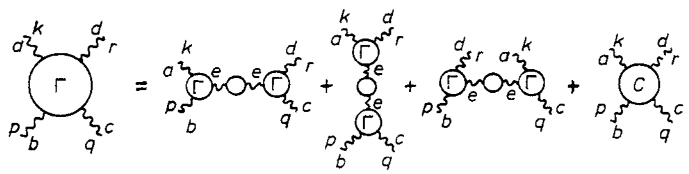

Figure 3. Decomposition of the amputated Green function into one-particle irreducible terms.

where M, N stand either for the vector indices μ, ν or the auxiliary fields coupling to $n\,.\,A$. Likewise,

$$k^{\kappa}C^{abcd}_{\kappa\lambda\mu\nu}(k,p,q,r)$$
$$=f^{abe}\Gamma^{cde}_{\lambda\mu\nu}(-q-r,q,r)+f^{ace}\Gamma^{dbe}_{\mu\nu\lambda}(-r-p,r,p)+f^{ade}\Gamma^{bce}_{\nu\lambda\mu}(-p-q,p,q) \tag{18}$$

and so on. For those readers unacquainted with the detailed checks of these identities, it is a useful exercise to go through them in perturbation theory as a preliminary for comprehending the fully dressed amplitude relations. In these checks, note the lowest-order expression $\Gamma=\Lambda$ and

$$C^{abcd}_{\kappa\lambda\mu\nu}=f^{abe}f^{cde}(\eta_{\kappa\nu}\eta_{\lambda\mu}-\eta_{\kappa\mu}\eta_{\lambda\nu})+f^{ace}f^{dbe}(\eta_{\kappa\lambda}\eta_{\mu\nu}-\eta_{\kappa\nu}\eta_{\mu\lambda})$$
$$+f^{ade}f^{bce}(\eta_{\kappa\mu}\eta_{\nu\lambda}-\eta_{\kappa\lambda}\eta_{\nu\mu}). \tag{19}$$

So much for the identities themselves. Next we write down the general spectral expansion

$$\Delta_{\mu\nu}(p)=\int \mathrm{d}W^2\frac{\mathscr{A}_{\mu\nu}(p)\alpha(W^2,p\,.\,n)+\mathscr{B}_{\mu\nu}(p)\beta(W^2,p\,.\,n)}{p^2-W^2+\mathrm{i}0}, \tag{20}$$

α, β even in $p\,.\,n$, where the kinematic factor $\mathscr{A}$ has been defined in (16) and

$$\mathscr{B}_{\mu\nu}(p)\equiv(\eta_{\mu\nu}-n_\mu n_\nu/n^2). \tag{16$'$}$$

The bare propagator corresponds to setting $\alpha\to\delta(W^2)$, $\beta\to 0$. We also know (Delbourgo 1978) the solution of the identity (12); namely, after contraction with $\Delta(p)$,

$$\Delta^{\lambda\lambda'}(p)\Delta^{\mu\mu'}(q)\Delta^{\nu\nu'}(r)\Gamma^{abc}_{\lambda'\mu'\nu'}(p,q,r)$$
$$=f^{abc}\int \mathrm{d}W^2\,\tfrac{1}{3}[\alpha(W^2,p\,.\,n)+\alpha(W^2,q\,.\,n)+\alpha(W^2,r\,.\,n)]$$
$$\times\frac{\mathscr{A}^{\lambda\lambda'}(p)\mathscr{A}^{\mu\mu'}(q)\mathscr{A}^{\nu\nu'}(r)\Lambda_{\lambda'\mu'\nu'}(p,q,r)}{(p^2-W^2)(q^2-W^2)(r^2-W^2)}$$
$$+f^{abc}\int \mathrm{d}W^2\,\tfrac{1}{3}[\beta(W^2,p\,.\,n)+\beta(W^2,q\,.\,n)+\beta(W^2,r\,.\,n)]$$
$$\times\frac{\mathscr{B}^{\lambda\lambda'}(p)\mathscr{B}^{\mu\mu'}(q)\mathscr{B}^{\nu\nu'}(r)\Lambda_{\lambda'\mu'\nu'}(p,q,r)}{(p^2-W^2)(q^2-W^2)(r^2-W^2)}. \tag{21}$$

We now have to insert (21) into (13) and determine the solution Γ^{abcd} up to transverse corrections. Having gone through electrodynamics, the strategy should be fairly clear: one observes that, aside from spectral averaging, the expression (21) is the sum of two terms, the first corresponding to a massive Yang–Mills particle and the second corresponding to a field with propagator $\mathscr{B}^{\mu\nu}/(p^2-W^2)$ also undergoing a Yang–Mills interaction. One can therefore look for the solution of the identity in lowest order for such a pair of model theories, and then simply sum over W^2 with the appropriate spectral function.

In the first model it is a fact (Delbourgo 1978) that for

$$D_{MN}=\begin{bmatrix}\dfrac{\mathscr{A}_{\mu\nu}(p)}{p^2-W^2}, & \dfrac{p_\mu}{p\cdot n}\\ \dfrac{p_\nu}{p\cdot n}, & 0\end{bmatrix} \tag{22}$$

$$D^{-1}_{MN}=\begin{pmatrix}(-\eta_{\mu\nu}+p_\mu p_\nu/p^2)(p^2-W^2), & n_\mu\\ n_\nu, & 0\end{pmatrix}. \tag{22'}$$

Before† integrating over W^2,

$$\begin{aligned}k^\kappa\Gamma^{abcd}_{\kappa\lambda\mu\nu}&(k,p,q,r)\\ &=f^{abe}f^{cde}[D^{-1}_{\lambda R}(p)D_{R\sigma}(q+r)\Lambda_{\sigma\mu\nu}(-q-r,q,r)]+\text{perms}\\ &=f^{abc}f^{cde}\{\Lambda_{\lambda\mu\nu}(\,,q,r)+[D^{-1}_{\lambda R}(p)-D^{-1}_{\lambda R}(q+r)]D_{R\sigma}(q+r)\Lambda_{\sigma\mu\nu}(\,,q,r)\}\\ &\quad+\text{perms}\\ &=f^{abe}f^{cde}\{\Lambda_{\lambda\mu\nu}(\,,q,r)+[D^{-1}_{\lambda\rho}(p)-D^{-1}_{\lambda\rho}(q+r)]D_{\rho\sigma}(q+r)\Lambda_{\sigma\mu\nu}(\,,q,r)\}\\ &\quad+\text{perms}.\end{aligned}$$

Since

$$D^{-1}_{\lambda\rho}(p)-D^{-1}_{\lambda\rho}(q+r)=k^\kappa\Lambda_{\kappa\lambda\rho}(k,p,-k-p)+W^2\left(\frac{(p+k)_\lambda(p+k)_\rho}{(p+k)^2}-\frac{p_\lambda p_\rho}{p^2}\right),$$

Γ can be factorised‡ in this model as follows:

$$\begin{aligned}\Gamma^{abcd}_{\kappa\lambda\mu\nu}&(k,p,q,r)\\ &=C^{abcd}_{\kappa\lambda\mu\nu}(k,p,q,r)+[f^{abe}f^{cde}\Lambda_{\kappa\lambda\rho}(k,p,\,)D^{\rho\sigma}(q+r)\Lambda_{\sigma\mu\nu}(\,,q,r)+\text{perms}]\\ &\quad+\frac{W^2n_\kappa}{k\cdot n}\left[f^{abe}f^{cde}\left(\frac{(p+k)_\lambda(p+k)_\rho}{(p+k)^2}-\frac{p_\lambda p_\rho}{p^2}\right)D^{\rho\sigma}\Lambda_{\sigma\mu\nu}(\,,q,r)+\text{perms}\right]\end{aligned} \tag{23}$$

whereupon multiplication by $D^{\kappa\kappa'}(k)$ eliminates the n_κ piece. The only thing that needs correcting is the spectral averaging and the lack of *full* Bose symmetry. This can be

† In the following we have adopted the shorthand $\Lambda(\,,q,r)$ to mean $\Lambda(-q-r,q,r)$ and $\alpha(\,,p_0)$ to mean $\alpha(W^2,p_0)$.

‡ We have chosen to factorise the last term of (23) in the form given, $n_\kappa W^2\{\,\}/n\cdot k$, rather than in the form $P_\kappa W^2\{\,\}/P\cdot k$—where P is a typical momentum in the diagram—by analogy with the earlier work on QED. The second kind of factorisation would not vanish upon contraction with $D^{\kappa'\kappa}$ or n^κ and would cause spurious covariant kinematic singularities, spurious because they would be an artifact of the factorisation and not connected with dynamical particle propagation poles.

achieved by writing, for the first term and the right-hand side of (21),

$$\tfrac{1}{3}[\alpha(\,,q_0+r_0)+\alpha(\,,q_0)+\alpha(\,,r_0)]$$
$$=\tfrac{1}{18}[3\alpha(\,,p_0)+3\alpha(\,,q_0)+3\alpha(\,,r_0)+3\alpha(\,,k_0)$$
$$+2\alpha(\,,p_0+q_0)+2\alpha(\,,p_0+r_0)+2\alpha(\,,r_0+q_0)]$$
$$+k^{\kappa}(n_\kappa/k\,.\,n)\tfrac{1}{18}[3\alpha(\,,q_0)+3\alpha(\,,r_0)-3\alpha(\,,p_0)-3\alpha(\,,k_0)$$
$$+4\alpha(\,,p_0+k_0)-2\alpha(\,,q_0+k_0)-2\alpha(\,,r_0+k_0)]$$

and factorising the last part† as indicated (similarly for the other terms on the right of (18)).

In the second model one may carry out the analogous steps except that the spectral function is β and the kinematic factor is $\mathscr{B}$ of (16′). When all the steps are dutifully followed, one obtains the sum total,

$$(\Delta(k)\Delta(p)\Delta(q)\Delta(r)\Gamma)^{abcd}_{\kappa\lambda\mu\nu}(k,p,q,r)$$
$$=\int \mathrm{d}W^2\,\bar{\alpha}(W^2,kpqr)\frac{\mathscr{A}^{\kappa'}_{\kappa}(k)\mathscr{A}^{\lambda'}_{\lambda}(p)\mathscr{A}^{\mu'}_{\mu}(q)\mathscr{A}^{\nu'}_{\nu}(r)}{(k^2-W^2)(p^2-W^2)(q^2-W^2)(r^2-W^2)}$$
$$\times\Big(C^{abcd}_{\kappa'\lambda'\mu'\nu'}+f^{abc}f^{cde}\Lambda_{\kappa'\lambda'\rho}(k,p,-p-k)\frac{\mathscr{A}^{\rho\sigma}(p+k)}{(p+k)^2-W^2}\Lambda_{\sigma\mu'\nu'}(-q-r,q,r)$$
$$+\text{two cyclic perms of } p\lambda'b,\ q\mu'c,\ r\nu'd\Big)$$
$$+(\text{identical expression with } \alpha\to\beta,\ \mathscr{A}\to\mathscr{B}). \qquad (24)$$

In (24) C and Λ are the bare vertices (14) and (19) of Yang–Mills theory, and the average‡ spectral function is

$$\bar{\alpha}(W^2,kpqr)=\tfrac{1}{18}[3\alpha(W^2,p\,.\,n)+3\alpha(W^2,q\,.\,n)+3\alpha(W^2,r\,.\,n)+3\alpha(W^2,k\,.\,n)$$
$$+2\alpha(W^2,p\,.\,n+q\,.\,n)+2\alpha(W^2,q\,.\,n+r\,.\,n)+2\alpha(W^2,r\,.\,n+p\,.\,n)]. \qquad (25)$$

The full Bose symmetry of the answer is readily verified by inspection. We emphasise once again that the Green function (24) is the only one which matters so far as solving the Dyson–Schwinger equations is concerned, and that it is linear in the spectral function. This eases considerably the task of finding gauge technique solutions for the propagator, the subject of a separate investigation. It is not true that the amputated Green function Γ has as simple a structure as (24); on the contrary, there are numerous n-dependent terms of the type exhibited in (6) and (23), so the reader should be warned of the dangers in deleting the factors $\mathscr{A}/(p^2-W^2)$ and $\mathscr{B}/(p^2-W^2)$ from (24) as a method of deducing Γ—it is plainly wrong.

† As $k\to 0$ in the last bracket, we meet the non-zero expression $[\alpha(\,,q_0)+\alpha(\,,r_0)+\alpha(\,,p_0)-3\alpha(\,,k_0)]$ which multiplies

$$f^{abe}f^{cde}\Lambda_{\lambda\mu\nu}(\,,q,r)+f^{ace}f^{dbe}\Lambda_{\mu\nu\lambda}(\,,r,p)+f^{ade}f^{bce}\Lambda_{\nu\lambda\mu}(\,,p,q)+k\,.\,\Lambda D\Lambda \text{ terms.}$$

This last combination of terms does vanish as $k\to 0$, as one can explicitly see from (18), and the factorisation is therefore analogous to the final part of (23).

‡ The factors in front of the spectral functions in (7), (11) and (25) can be ascertained by drawing all the pole contributions to the Green function and attaching unit weight to each charged line arriving at every vertex and summing over all distinct channels.

To summarise, we have solved the four-point gauge identities (4) and (13) for electrodynamics and chromodynamics, in terms of the complete propagators (1), (8) and (20) for the axial gauge. Our method generalises easily to higher-point Green functions, and we have only resisted the temptation of quoting the answers because of the complex and lengthy character of the expressions, although they are well understood in principle. They are anyway not needed in first gauge approximation.

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References

Anishetty R, Baker M, Kim S K, Ball J S and Zachariasen F 1979 *Phys. Lett.* **86B** 52
Ball J S and Zachariasen F 1978 *Nucl. Phys.* B **143** 148
Bargadda U 1979 *Stanford University Preprints*
Delbourgo R 1978 *J. Phys. A: Math. Gen.* **11** 2057
—— 1979a *Nuovo Cimento* A **49** 484
—— 1979b *J. Phys. G: Nucl. Phys.* **5** 603
Delbourgo R and Phocas-Cosmetatos P 1979 *J. Phys. A: Math. Gen.* **12** 191
Johnson K 1960 *Ann. Phys., NY* **10** 536
Salam A 1963 *Phys. Rev.* **130** 1287